**Homework 14**



**P23.2.7** Determine *vO*(*t*) in Figure p23.2.7 if  V.

Solution:

**Solution:** The T-circuit is shown. It follows that   ; ; hence,   . The IFT is V.



**P23.3.6** If *i*(*t*) = 2sinc(4*t*), determine: (a) the frequency band starting at *ω* = 0 that contains half the energy of *i*(*t*), (b) the energy dissipated in a 2 Ω resistor due to *i*(*t*) applied over all time.

**Solution:** The FT of *Im*sinc(4*t*) is , and the FT of 2sinc(4*t*) is .

(a) The frequency band 0-4 rad/s contains all the energy of the signal. It follows that the band 0-2 rad/s contains half the energy.

(b) |*F*(*jω*)|2 = (*π*/2)2 = *π2*/4. The energy dissipated in a 1 Ω resistor is (1/*π*)(*π2*/4)×4 = *π* J. The energy dissipated in 2 Ω resistor is 2*π* J.

**P23.3.8** If *u*(*t*) V in Figure P23.3.8, what percentage of the 1 Ω energy of *vO* is in the frequency range rad/s if: (a) *R* = 2 Ω; (b) *R* = 4 Ω.



**Solution:** Let ; hence, ; ; ; = . The energy is = . The total energy is = = ; the energy between 0 and 2 rad/s is: = . The ratio is therefore .

(a) When *R* = 2 Ω, *α* = 0.5, and the percentage is 38.5%.

(b) When *R* = 4 Ω, *α* = 0.25, and the percentage is 44.0%.

**P24.1.15** (a) Determine the *h* parameters and *g* parameters of the circuit in Figure P24.1.14 from the definition of these parameters. (b) Verify that the *h* and *g* matrices are the inverse of one another.



**Solution:** (a) The T-equivalent circuit is shown. With port 2 short circuited, **V1** = (-*jω* + **)**I1****I1** S; hence, *h*11=  Ω; *h*21= ; with port 1 open circuited, *h*12; *h*22S. With port 2 open circuited, *g*11=S; *g*21=  =. With port 1 short circuited, *g*12; *g*22Ω.



(b) Consider Δ*h* = *h*11*h*22 – *h*12*h*21 =  +  =  +  +  =  +  = ; ; *g*11 = ×= , as before; *g*12 = , as before; *g*21 =   , as before; *g*22 = ×, as before.

**P24.1.24** The two two-port circuits are identical and have *z*11 = *z*22 = 2 Ω, and *z*12 = *z*21 = 1 Ω. If *V*o = *ρIsrc* + *αVsrc*, determine *ρ* and *α*.



**Solution:** The second *z*-parameter equation for the first circuit is:

*V*2 = *Isrc* + 2*I*2 (1)

The z-parameter equations for the second circuit are: *V*2 + *Vsrc* = + 0 (2)

and *Vo* = + 0 (3)

where *I*2 = -. Substituting for *I*2 in Equation (1): *V*2 = *Isrc* –  (4).

Substituting for *V*2 from Equation (4) and for  from Equation (3) in Equation (2), *Isrc* – 2*Vo* + *Vsrc* = 2*Vo*. This gives, 4*Vo* = *Isrc* + *Vsrc*, so that *ρ =*1/4 Ω*.*

**P24.2.2** Determine *K* so that the circuit in Figure P24.2.2 is symmetric.



**Solution:** With the input open circuited and a voltage applied to the output terminals, *I*1 = 0, and *V*1 = 2*I*2, so that *z*12 = 2. With the output open circuited, *I*2 = 0, and *V*2 = 2*I*1 + *KI*1 = (*K* + 2)*I*1; hence *z*21 = *K* + 2. For the circuit to be reciprocal, *z*12 = *z*21, so that *K* = 0. Once the dependent source is set to zero, *z*11 = *z*22 = 12 Ω, and the circuit is symmetric.

**P24.2.4** A symmetric two-port circuit has an open-circuit input impedance of (1 – *j*) Ω, and an open-circuit transfer impedance, that is *V*2/*I*1, of 1 Ω. Determine the load current if a load impedance of *j* Ω is connected to one port and a 10∠0° V source is connected to the other port.



**Solution:** The transfer impedance of 1 Ω is the shunt impedance in the T-equivalent circuit. The series impedance is then *z*11 – *z*12 = (1 – *j*) – 1 = -*j* Ω. The T-equivalent circuit becomes as shown. The impedance -*j* + *j* = 0, so that *IL* = *IS* = 10∠0°/-*j* = *j*10 A.



**P24.2.6** The two-port circuit in Figure P24.2.6 is described in the *s*-domain by the equations:

*I*1 = 2*sV*1 – *sV*2,

*I*2 = -*sV*1 + 2*sV*2

(a) Show that the two-port circuit is symmetric; (b) Determine the transfer function *H*(*s*) = *Vo*/*Vsrc*.

**Solution:** (a) If *I*1 is interchanged with *I*2, and *V*1 is interchanged with *V*2, the first equation becomes: *I*2 = 2*sV*2 – *sV*1, which is the same as the second equation. The second equation becomes *I*1 = -*sV*2 + 2*sV*1, which is the same as the first equation. Since the same equations apply when the input and output variables are interchanged, the circuit is symmetrical.

(b) KCL at the input terminals gives: . Substituting for *I*1 from the first two-port circuit, and setting *V*2 = *V*o, gives: 2(*s* + 1)*V*1 = *Vsrc* + (*s* + 1)*V*o. KCL at the output terminals gives: , or, (*s* + 1)*V*1 = (3*s* + 1)*V*o. It follows that *Vsrc* + (*s* + 1)*V*o = 2(3s + 1)*V*o, or *Vsrc* = (5*s* + 1)*V*o, so that .

